Refining Metcalfe's Law for Digital Blockchain Network Valuation

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Abstract: We introduce an extended valuation model for the Ethereum Network, drawing from interventions based on Metcalfe's Law to account for network effects and building upon existing standard models of digital assets. The model incorporates an exponential growth decay equation to dynamically adjust circulating supply, considering network upgrades. We employ non-linear regression to capture complex patterns in demand and supply functions at smaller time scales while adhering to the overall trend. Our model evaluates network quality and utility beyond mere size, enhancing predictive accuracy. Empirical results demonstrate that our model offers improved precision over both standard and previous Metcalfe's law models.

Keywords: cryptocurrency, digital blockchain networks, ethereum network, metcalfe's law, network effects

1. Introduction

In the modern digital landscape, where blockchain technologies and networks have emerged as transformative forces in global finance and decentralized systems, while forging a novel market that constitutes a unique asset class for investment portfolios, the accurate valuation of these networks has become an essential consideration for investors, policymakers, and researchers. Digital blockchain networks, such as Ethereum, represent not only innovative platforms for decentralized applications but also complex economic ecosystems whose value is influenced by network effects (user adoption), quality (cost efficiency and technological upgrades), and utility (transactional performance). Traditional fundamental valuation approaches, often borrowed from conventional financial models, have proven inadequate in capturing the unique dynamics of these networks, underscoring the need for more robust frameworks that account for both network growth and economic factors. In light of the approvals of Ethereum ETFs, the incorporation by public companies of Ethereum into their corporate treasuries, and the recent advancements in the regulatory framework of blockchain technology, valuation models have attained heightened significance in guiding retail and institutional investment decisions.

Significant contributions have been made in the field of digital asset valuation, with standard models in the blockchain industry having been proposed by Refs. [1−3]. These models, primarily modifications of Metcalfe's Law, which posits that the value of a network grows proportionally to the square of its users, have demonstrated explanatory power for networks like Bitcoin by incorporating network effects and supply dynamics. Nonetheless, their application to more evolving platforms like Ethereum, which features smart contracts, layer-2 scaling solutions, and protocol upgrades such as EIP-1559 and the transition to Proof of

Stake (PoS), reveals shortcomings. Assumptions in standard models are limited in addressing Ethereum's non-linear growth dynamics, quality improvements, and utility beyond simple user counts, thereby reducing predictive accuracy.

To address these challenges, Metcalfe's Law, when refined with elements like exponential growth decay equation and quality metrics, can better model the interplay of demand, supply, and network maturity in digital blockchain networks. This research builds upon prior adaptations by integrating non-linear regression techniques and Artificial Neural Networks (ANNs) to capture complex patterns at smaller time scales, while extending the framework to include quality and utility variables as proposed by Refs. [4, 5]. In a collaborative effort, we thoroughly analyze these proposed standard models, identify the most effective approach for valuing the Ethereum Network, and further augment it with empirical enhancements. This model demonstrates a valuable impact on the explanatory power of the price of the Ethereum Network. Hence, the aim of this research is to propose an accurate predictive model for the price of the Ethereum network and to assess economic aspects, including techniques for measuring resource-use efficiency and cost, through a multi-factor model that incorporates network size, quality (TCPU), and utility (TPPF). This model not only refines Metcalfe's Law for blockchain networks contexts but also provides insights for broader applications in Layer 1 public blockchain networks, ultimately contributing to more informed investment decisions and theoretical advancements in digital asset economics.

2. Literature Review

In Ref. [1], one of the pioneers of contributing to the standard model of digital assets, has shown that the value of Bitcoin, Ethereum and Dash Network appears to follow Metcalfe's Law deriving growth with Netoid/Sigmoid function. Alabi [1] proposed a new network model Eq. (3) derived by the growth rate equation expressed in Eq. (1) and the exponential function as in Eq. (2), whereby efficiency increases. Alabi [6] proved again that both models fit the data well in the Bitcoin Network and states that future research will most likely be motivated by the growth of digital assets.

$$N(t) = N_0 e^{vt} \tag{1}$$

$$V(N) = Ce^{\lambda \bar{N}^m} \tag{2}$$

$$V(N) = Ce^{\lambda\sqrt{\bar{N}}} = Ce^{\sqrt{\gamma \bar{N}}} \tag{3}$$

The work of Ref. [2], which demonstrates that Bitcoin's medium-to-long-term price follows Metcalfe's Law, also contributes to the application of this law in valuing Bitcoin. Furthermore, it incorporates the Gompertz sigmoid function into the model as in Eq. (4) to account for the inflationary effects related to the creation of new bitcoins (b=numbers of bitcoins created and B = is the total of number of bitcoins), which consequently leads the Bitcoin price to significantly fit Metcalfe's law extension model. Peterson [2] expressed in Eq. (5) and the final regression model uses a generalized difference equation to mitigate for autocorrelation, as is illustrated in Eq. (6).

$$b_t = b_{t-1} \times \ln\left(\frac{B}{b_{t-1}}\right) \tag{4}$$

$$V = A \times \left[\frac{n(n-1)}{2} \times \frac{1}{b_t}\right] \tag{5}$$

$$\ln(Y_t) - p\ln(Y_{t-1}) = a_0(1-p) + \beta_0[\ln(X_t) - p\ln(X_{t-1})] + u_t \tag{6}$$

Subsequently, Vliet [3] augmented the model proposed in Ref. [2] and made two key contributions to a new model of Bitcoin's market capitalization. The first integrates logistic sigmoid function as in Eq. (7), which follows diffusion of innovations capturing growth rates and user's parameters. The second incorporates the number of bitcoins enabled by bounded exponential growth equation capped to 21 million bitcoins, as is presented in Eq. (8). Vliet [3] proposed model follows a linear regression expressed in Eq. (9) that fits the empirical data well and opens the door to further research on embedding economic variables into theoretical models.

$$\hat{n}_t = \frac{N}{1+\phi e^{-vt}} \tag{7}$$

$$\hat{b}_t = B(1 - e^{-\eta t}) \tag{8}$$

$$\ln(V_t) = 0 + \beta \left[\frac{\ln(\hat{n}_t)}{\ln(\hat{b}_t)}\right] + \varepsilon_t \tag{9}$$

Hove [4] pointed out that the value of a social network may be driven not only by its size, but also by increases in the variety and quality of the services offered. Hence, he contributes to Metcalfe's model by adding a quality variable to the regression model, as is expressed in Eq. (10). As a proxy of the quality indicator of the network, he incorporates "cost per user" or "Cost Per Node" (CPN) for both Facebook and Tencent, with sizes determined by Metcalfe's Law (n^2). Hove [4] concluded that these extensions only strengthen Ref. [7] conclusions, Metcalfe's law now outperforms the other laws even more clearly.

$$V = a \times SIZE + b \times QUALITY \tag{10}$$

Upon evaluation of these standard models and their extensions, which demonstrate significant explanatory power for digital assets, primarily Bitcoin prices, these models are subsequently assessed for their applicability to the valuation of the Ethereum Network.

3. Methodology and Baseline Model

Data from the Ethereum Blockchain Network were obtained from glassnode.

3.1. Model Supply

Firstly, similar to Ref. [3] in modeling the number of coins, the circulating supply of Ether, s_t, behaves based on the trend of exponential growth decay over time although it is characterized by a non-bounded exponential growth equation to model the supply of the Ethereum network, as shown in Eq. (11). The model incorporates cumulative net issuance i as the variable to model the supply for the Ethereum Network and capture the changes to the constitutional formulation of ether coins. Cumulative net issuance is defined as the cumulative issuance of ether coins subtracted from the cumulative ether coins burned. The parameters α, b, and η are determined by fitting the equation. The selected supply model appears to provide a balanced method for incorporating cumulative net issuance. This model captures both a linear trend $(\alpha + b \times i)$and an exponential component $(i \times e^{-\eta t})$, accounting for non-linear behavior due to factors such as EIP-1559 and the PoS upgrade. It seems to consider the effects of time on past and future upgrades, with the exponential decay factor $(e^{-\eta t})$ allowing for the diminishing effect of time on the supply growth rate. As a result, the model is better equipped to capture the varying rate of change in Ethereum's circulating supply, effectively representing inflation and deflation at different stages.

$$\check{s}_t = \alpha + b \times \mathrm{i} \times \mathrm{e}^{-\eta t} \tag{11}$$

3.2. Model Demand

Secondly, Alabi [1] proposed network model, as shown in Eq. (12), to model the demand and quantify the network effects value for the Ethereum Network. The model is based on the principle that the growth in the number of users, N, is governed by an exponential function. In this model, $\overline{N}$ incorporates a 30-day average filter to account for fluctuations in the number of users over time. The variable d_t represents the ether's price, while the parameters of C, λ, and m are determined by fitting the Eq. (12). The variables d_t and, $\overline{N}$ are transformed into lognormal values. Log transformation is a common practice when dealing with currency returns, as Ref. [2] states. Alabi [1] proposed network model approach quantifies the network effects value by incorporating active addresses, acknowledging that digital assets do not have fixed network effects values like those in Metcalfe's law. This indicates that the model supports a "not pre-ordained exponent" in the same way that N^2, as [1] states. Importantly, according to Ref. [1], this "not pre-ordained exponent" feature allows the model to be utilized even as the network reaches maturity. This suggests that the diffusion of innovations is not an imperative for the model's application. Given that the Ethereum Network has a layer 1 and a growing base of layer 2 solutions, Alabi [1] proposed network model appears to capture the network effect value more effectively.

$$\check{d}_t = Ce^{\lambda \bar{N}^m} \tag{12}$$

3.3. Regression

Lastly, akin to the model that [2] proposes, the logarithm of Ethereum's price, y_t as shown in Eq. (14), follows a regression pattern equation albeit nonlinear regression equation to model the demand and supply for Ethereum Network. The variables V_t and $\check{s}_t$ are transformed into lognormal values, while the variable $\check{d}_t$ is already in lognormal form. By dividing the demand model Eq. (12) by the supply model Eq. (11), we develop the ratio of demand and supply x_t as demonstrated in Eq. (13). This concept is a fundamental principle in economics, known as the law of supply and demand. Peterson [2] applied this approach, stating that "bitcoin's equilibrium value is based solely on factors relating to supply (number of bitcoins) and demand (number of wallets)." The result of [2] model is Metcalfe's law (demand model), adjusting for the creation of new bitcoins (supply model) over time, using the Gompertz sigmoid function as expressed in Eq. (5). Similarly, in our model, we apply the law of supply and demand through the network effects value model that Ref. [1] develops, adjusting by the circulation supply using an exponential growth decay function, as illustrated in Eq. (13). In the study by Ref. [8], a comparison table of previous work in artificial neural network models and simple & multiple regression models for cryptocurrency prediction is presented. It is noteworthy that artificial neural network models appear to outperform regression models in this context. Consequently, we deploy the baseline model, a nonlinear regression Eq. (15), using Artificial Neural Network (ANN) models, which are proficient at recognizing complex nonlinear patterns, particularly for capturing smaller time scales. This approach appears to enable a more effective indirect integration of past (EIP-1559 and POS) and future upgrades of the Ethereum Network. The non-linear function f and the parameters β are determined by the architecture and training of our specific ANN model.

$$x_t = \frac{ln(\check{d}_t)}{ln(\check{s}_t)} \tag{13}$$

$$y_t = ln(V_t) \tag{14}$$

$$y_t = f(x_t, \beta) + \epsilon_t \tag{15}$$

We represent, evaluate, and optimize the architecture of the machine learning model based on the Algorithm 1[1], drawing intervention based upon the study conducted by Ref. [9]. Zhang *et al.* [9] asserted that the Weighted and Attentive Memory Channels model, also known as WAMC excels at modeling the non-linear correlations between cryptocurrencies, even when considering a single cryptocurrency such as ETH. Furthermore, their study demonstrated that the WAMC model outperforms all commonly used baseline

[1] Refer to Appendix A for a summary.

models. Given that x_t and y_t demonstrate a strong correlation and exhibit a degree of non-linearity, we propose a Hybrid Recurrent-Convolutional Neural Network with an Attention Mechanism model[2].

Considering our objective to develop a model that accurately captures smaller time scales and follows the trend when zoomed out at that scale, we initially focus our optimization efforts on the Algorithm 1[3]. Specifically, we adjust the window size, batch size, and epochs. Subsequently, we fine-tuned the remaining parameters and hyperparameters to further improve the model's performance, as evaluated by R-squared and Root Mean Square Deviation (RMSD) values. In the upcoming sections, we discuss the details of our final model optimization.

4. Quantitative Results

In the supply model, we fit the number of ether coins over time, s_t, as described in Eq. (11), to daily data points. The data points selected for this study encompass the period from September 1, 2015, through December 7, 2022. Fig. 1 (and as well Fig. 2 and Fig. 3) exhibit the empirical data illustrated by a solid line, while the model is illustrated by a dashed line. Table 1 illustrates the curve fit output of the supply model, specifically the circulation supply. The R-squared value is significantly high at 0.99, and the RMSD at 224.52, indicating a strong fit.

Table 1. Output of curve fit applied to supply model

Curve fit statistics	
Optimized a	72,910,739.0934
Optimized b	0.9999
Optimized n	−1.5390E-09
R-squared	0.9999
RMSD	224.5190
Observations	2,655

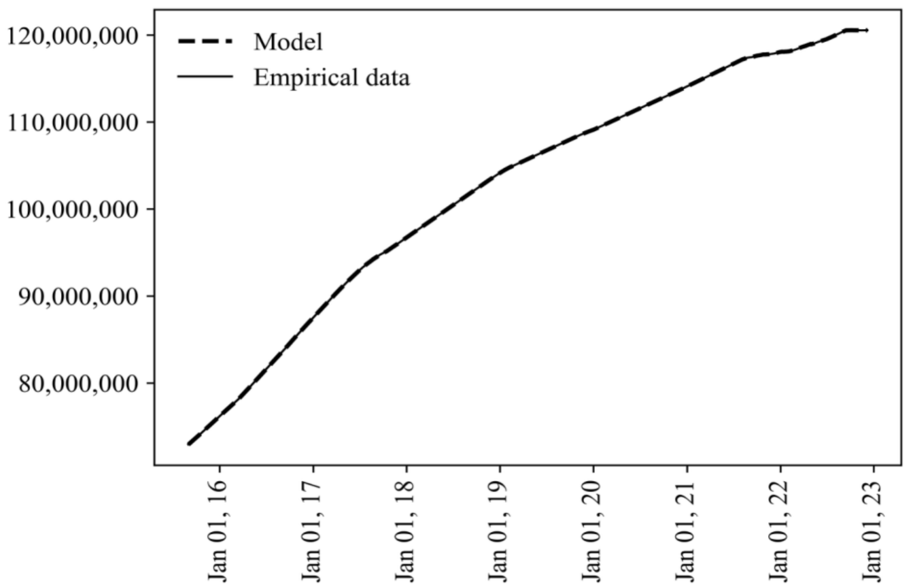

Fig. 1. Supply model fit to the number of ether coins.

In the context of the demand model, we utilize the network model that Ref. [1] proposes Eq. (12) to fit the

[2] The layers of this model are described in Appendix A.2.
[3] See Appendix A.3 for parameter configurations of each learning algorithm component.

log of price of Ethereum Network. The empirical data and the model are shown in Fig. 2. The optimized values of C, λ, and m which maximize the R-square and minimize RMSD are displayed in Table 2 along with the curve fit output. The R-squared value is notably high at 0.91, and RMSD at 0.63, indicating a robust fit.

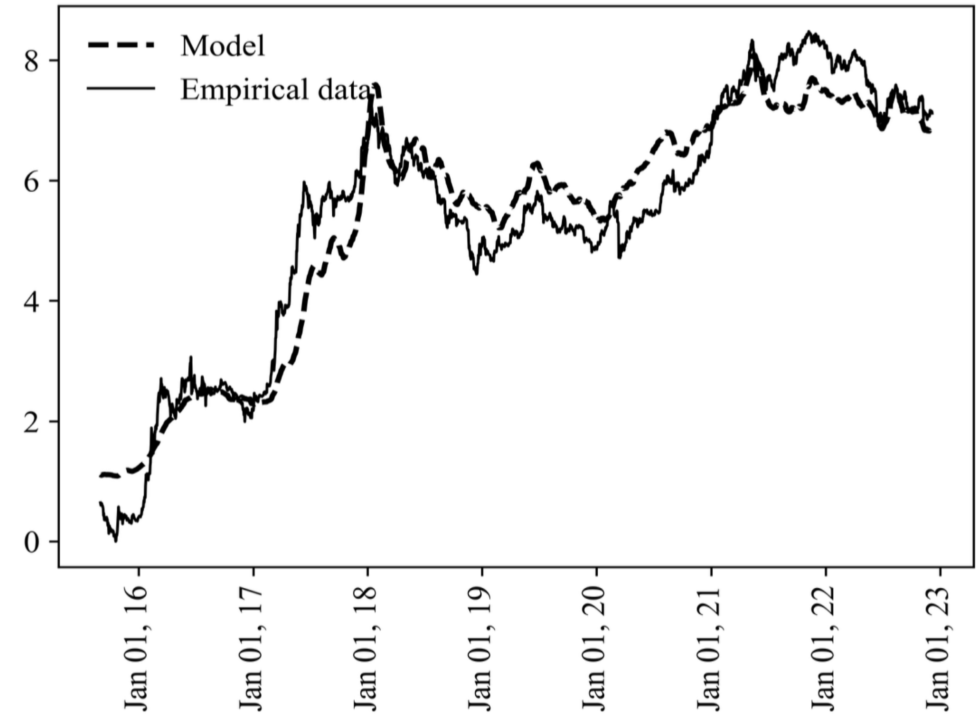

Fig. 2. Demand model fit to log of price ethereum network.

Table 2. Output of curve fit applied to demand model

Curve fit statistics	
Optimized m	1.2520
Optimized λ	0.1444
Optimized C	0.1930
R-squared	0.9129
RMSD	0.6270
Observations	2,655

We deploy the supply and demand model, the baseline model, which is our specific ANN model as described in detail in Appendices A.1 to A.3. The model was trained on the dataset, as illustrated in Fig. 3, where the empirical data is represented by the solid line and the ANN model's predictions are represented by the dashed line. Table 3 illustrates the performance output and parameters settings[4] obtained from the ANN model's predictions for the demand and supply model of the Ethereum Network.

The ANN model exhibits a R-squared value that is distinctly high, reaching 0.98, and RMSD at 0.27, implying that the final supply and demand model appears to have strong explanatory power. It is worth mentioning that a train-test split was implemented to help mitigate the risk of model overfitting. The performance metrics, R-squared and RMSD, remained nearly identical, regardless of the application of the train-test split. This consistency could suggest that the model is effectively generalizing and not merely memorizing the training data (overfitting). As a result, the ANN model's predictions, as displayed in Fig. 3, demonstrate high accuracy

[4] Further parameter settings are specified in Appendix A.3.

with relatively low prediction errors.

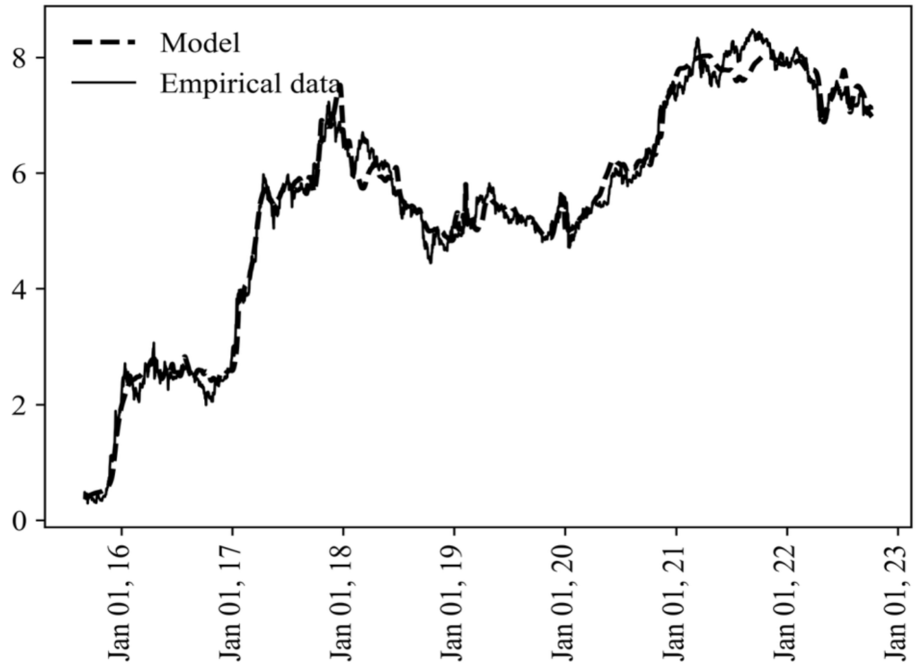

Fig. 3. Demand and supply model fit to log of price ethereum network.

Table 3. Output of ANN model and parameters settings (demand and supply model).

Category	**Parameter**	**Value**
Model performance statistics	R-squared	0.9801
	RMSD	0.2763
	Observations	2.655
Model compilation & training	Window size	60
	Number of epochs	170
	Batch size	90
	Loss function	MSE
	Optimizer	Adam
	Learning rate	0.001

Additionally, we can highlight from the supply and demand model depicted in Fig. 3 that there are periods where the price of the Ethereum Network deviates from the model's predictions. These deviations could be attributed to various factors such as price manipulation or the influence of macroeconomic variables like changes in Fed Fund rates, inflation rates, or unemployment rates. Peterson [10] highlighted the presence of anomalies in Bitcoin prices during the years 2013, 2017, and 2019 through Benford analysis. Furthermore, Peterson [10] stated that fraudulent manipulation of Bitcoin's price has occurred at certain points since its inception in 2010. Considering that the Ethereum Network belongs to the same asset class, it is plausible that similar factors contribute to the periods of misalignment between the supply and demand model and the price of the Ethereum Network. At this critical juncture, it becomes noticeable that the ANN model is useful in fishing out periods where manipulation is dominant over the intrinsic forces of demand and supply that the model appears to capture quite well.

5. Discussion

To visualize the data, Fig. 4 demonstrates a high correlation between the variable x_t and the variable y_t from Eq. (15), which is 0.95. Furthermore, the relationship between these variables exhibits some degree of non-linearity. Therefore, our final model architecture proves to be a superior approach compared to simple linear regression, such as the approach used by Ref. [2], due to the presence of non-linear patterns between the variables. Additionally, incorporating similarities from the model architecture proposed by Ref. [9] proves beneficial, given the high correlation between the variables.

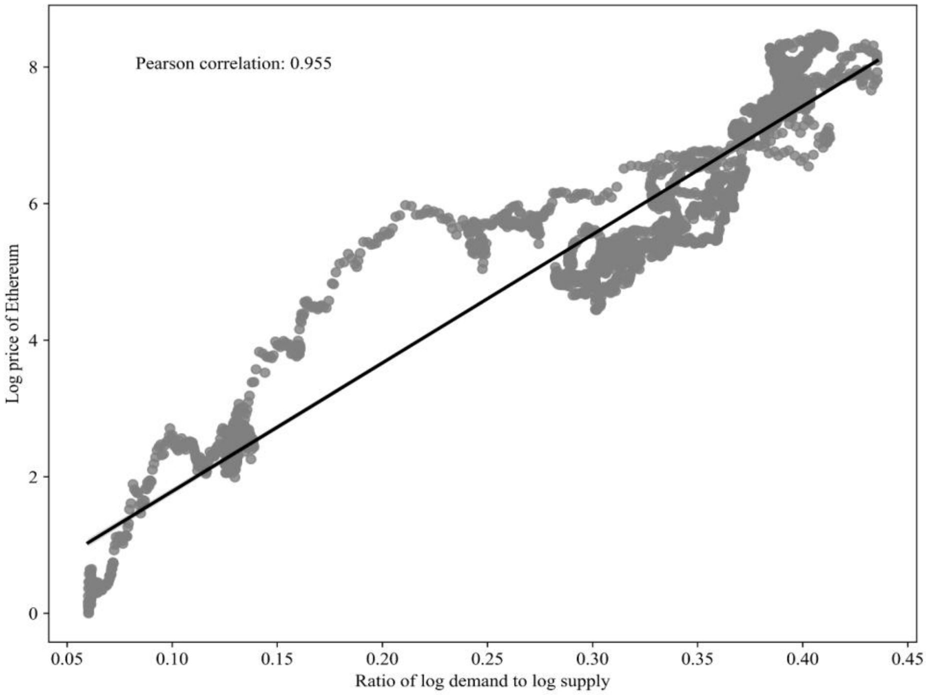

Fig. 4. Regression line of log price of Ethereum and ratio of log demand to log supply.

5.1. Extensions and Baseline Model

In this section, we explore extensions of the baseline model with the objective of identifying key insights and addressing criticisms of Metcalfe's Law, thereby contributing empirical enhancements to the foundational understanding of Metcalfe's Law. This exploration is anchored in the seminal work of [4], whose research on quality variables has set a precedent for augmenting our baseline model by integrating critical independent variables associated with the quality and utility of the Ethereum Network into the regression framework. Hove [4] stated that the value of a social network is influenced not only by its size but also by the diversity and quality of services it offers. Furthermore, Hove [4] suggested that quality improvements can manifest as enhanced connection reliability or quicker response times during interactions. In alignment with these premises, it is reasonable to infer an increase in both the utility and quality of the network. Finally, [4] asserts, "safe to state that networks possess utility that is independent of their network size." Furthermore, "unpublished" [11] states that Metcalfe's Law has two key criticisms, and the one we do not directly address in our baseline model is that diminishing marginal returns, quality, or transaction volume are not accounted for. "Unpublished" [11] defines transaction volume as a function of network capacity and size.

Building upon insights from Metcalfe's Law as described by Ref. [4], our study expands the analytical framework to include quality variables such as Transaction Cost Per User (TCPU). It is essential to acknowledge that while the cost per user may reflect aspects of network quality independent of size, no metric is entirely isolated. The TCPU, alongside transaction throughput, latency, and capacity, must be analyzed collectively for a holistic understanding of a blockchain network's quality and performance. Consequently, we introduce a utility variable, Transaction Processing Performance Function (TPPF), denoted by the arctan function as proposed by Ref. [5], enhancing network quality understanding.

5.2. First Extension

The first extension incorporates the $TCPU_t$ as a quality metric within the Ethereum Network, capturing qualitative service changes. This approach, inspired by Ref. [4] intervention-based analysis, adapts the centralized networks' cost per user metric for the Ethereum blockchain's context. We use the ratio of g (gas expenditure in USD) to n (daily active users) to approximate the average, serving as a quality assessment proxy as is illustrated in Eq. (16).

$$TCPU_t = \frac{g}{n} \tag{16}$$

This variable suggests that focusing on the network's quality could lead to an increase in cost per user over time. According to Ref. [4] analysis, while Metcalfe's law primarily addresses the growth of a network's value with its size, there is a common misconception that it implies costs evolve linearly with network size. In reality, Ref. [4] differentiates between the systemic costs borne by network members and the operational costs of the network company. Ref. [4] indicates that aggregate costs, rather than increasing linearly with network size, are manifested through the interactions and transactions among users, which, in the context of Ethereum, are represented by transaction fees. Zhang *et al.* [7] found that for these two social networks, Tencent and Facebook, the linear-cost hypothesis is unrealistic, leading Ref. [4] to suggest that costs increase beyond linear growth.

The interpretation by Ref. [4] of increasing cost per node, as evidenced by Facebook and Tencent, signifies quality improvements, this increase implies that the network is delivering more value per user, which presumably incurs higher costs. Therefore, according to Ref. [4] underlying assumption, an increase in cost per node (costs rising faster than user growth), interpreted as costs increasing beyond linear growth, is seen as an indication of quality improvements, with the analysis specifically focusing on variable costs rather than including fixed costs.

Adopting these foundational premises for the Ethereum Network's, we anticipate an increase in TCPU in the short term, reflecting heightened quality expectations. This increase may result from users' willingness to pay more for perceived enhancements such as scalability, security, and decentralization. High network utilization may lead to increased transaction fees, reflecting the network's expanding utility and user engagement. Over time, the proliferation of decentralized applications is expected to enhance the network's utility and value proposition, justifying higher transaction costs. In the long term, we hypothesize a smooth upward trend toward the stabilization of TCPU, indicating enhanced network quality. This maturation signifies a network characterized by balanced transaction demand and block space competition. The introduction of EIP-4844 aims to reduce fees associated with Layer 2 rollups, potentially decreasing main network transaction costs. Thus, in the long term, EIP-4844 serves as an example of network quality in action.

In sum, utilizing Metcalfe's Law and Ref. [4] premises, we outline a life cycle of network quality evolution. In the short term, increases in TCPU indicate improving quality and escalating user costs, driven by heightened functional expectations and demand for block space. Conversely, in the long term, a smooth upward trend toward the stabilization of TCPU reflects a maturing, more efficient network. These dynamic highlights the evolving nature of the Ethereum network, shaped by user demands, technological advancements, and economic factors.

In line with Ref. [4] analytical methodology, this quality indicator, denoted as TCPU, is integrated into the baseline regression model as an independent variable. The cost per user metric is transformed into lognormal values, facilitating its incorporation into a non-linear regression analysis via ANN models[5], and encapsulated in Eq. (17), as well as within a linear regression framework, represented as Eq. (18). Here, $x_{1,t}$ corresponds

[5] Systematically presented in Appendix A.1

to the variable x_t from Eq. (13), $x_{2,t}$ to $TCPU_t$, and Y_t to the logarithm of the Ether price. This nuanced integration seems to enhance the model's predictive capability by including the TCPU variable. Consequently, the refined model more accurately captures the intricate variations in Ethereum's transaction cost dynamics, effectively delineating the cost-related implications for quality improvements.

$$Y_t = f(x_{1,t}, x_{2,t}, \beta) + \epsilon_t \tag{17}$$

$$Y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \epsilon_t \tag{18}$$

Fig. 5 provides a visual analysis of the network's quality, showing a consistent increase in TCPU on the Ethereum Network. This trend suggests an improvement in network quality and supports our reasonable assumption that users are willing to pay more to utilize the enhanced quality of the network and in anticipation of future improvements. This occurs alongside high network utilization, including demand for transactions and competition for block space, further reflecting the network's expanding utility and user engagement. Details of this are further explored in Fig. 6, which highlights the augmented utility and capacity. In Eq. (5), the transition from logarithmic values of −6 to +2 illustrates the range and scale of variation in the TCPU variable, from very low (yet positive) transaction costs per user to higher transaction costs.

The use of a logarithmic transformation aids in managing and visualizing the wide range; nonetheless, it does not imply that the actual costs are negative.

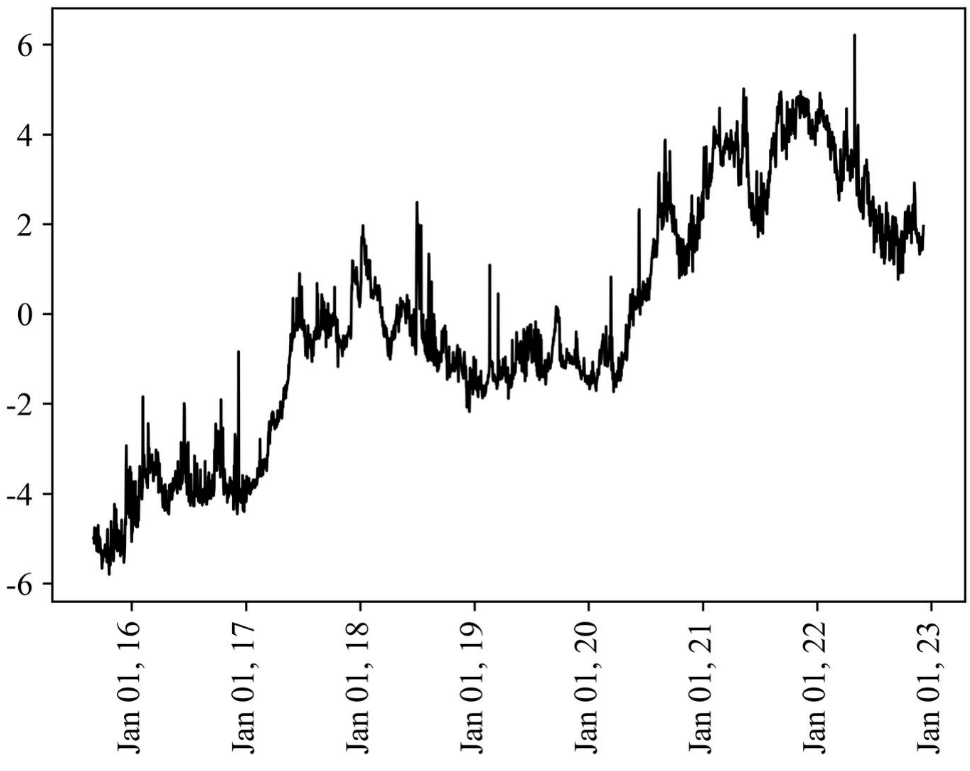

Fig. 5. Log TCPU over time.

5.3. Second Extension

The second extension incorporates the utility function $TPPF_t$ as proposed by Ref. [5] to model the utility variable, quantifying the efficiency of transaction processing capabilities per block over time within the Ethereum Network as is expressed in Eq. (19). This adaptation, although originally utilized for the

Transmission Control Protocol (TCP) of the Internet, is adapted within the unique context of the Ethereum Blockchain Network.

In adapting the utility function from its original context of network traffic management in TCP to the Ethereum blockchain, we utilize the product of X_r (the number of transactions per second) and B_t (the average block time in seconds) to approximate the average number of transactions per block. This product, analogous to the "flow rate" in Ref. [5] utility function for TCP network optimization, represents the utility of a block in terms of its transaction count. The utility function, governed by the arctan function, ensures a bounded output, preventing indefinite escalation of the utility indicator. The arctan function modulates utility growth, increasing more gradually as its input expands. A scaling factor, sf, is applied as a normalization constant within the utility function, with a value of $\sqrt{2}$. This value is derived from Ref. [5] reasoning.

$$TPPF_t = \frac{sf}{B_t} \times \arctan\left(\frac{X_r \times B_t}{sf}\right) \tag{19}$$

Mirroring the methodological approach of Ref. [4], the utility function, as derived by Ref. [5], is adeptly integrated into the baseline regression model as an independent variable. This integration is executed in a non-linear regression analysis utilizing ANN models, with the model architecture and encapsulated in Eq. (17), and within a linear regression framework as Eq. (18). Here, $x_{1,t}$ corresponds to the variable x_t from Eq. (13), $x_{2,t}$ to $TPPF_t$, and Y_t to the logarithm of the Ether price. This strategic incorporation appears to enhance the model's predictive capacity regarding Ethereum's price fluctuations by factoring in the utility function. As a result, the enhanced model adeptly captures the nuanced variations in Ethereum's utility, effectively delineating transaction throughput and its implications for transaction execution at various stages. The utility function emerges as a crucial metric for assessing both the efficiency and effectiveness of the Ethereum network's TPPF.

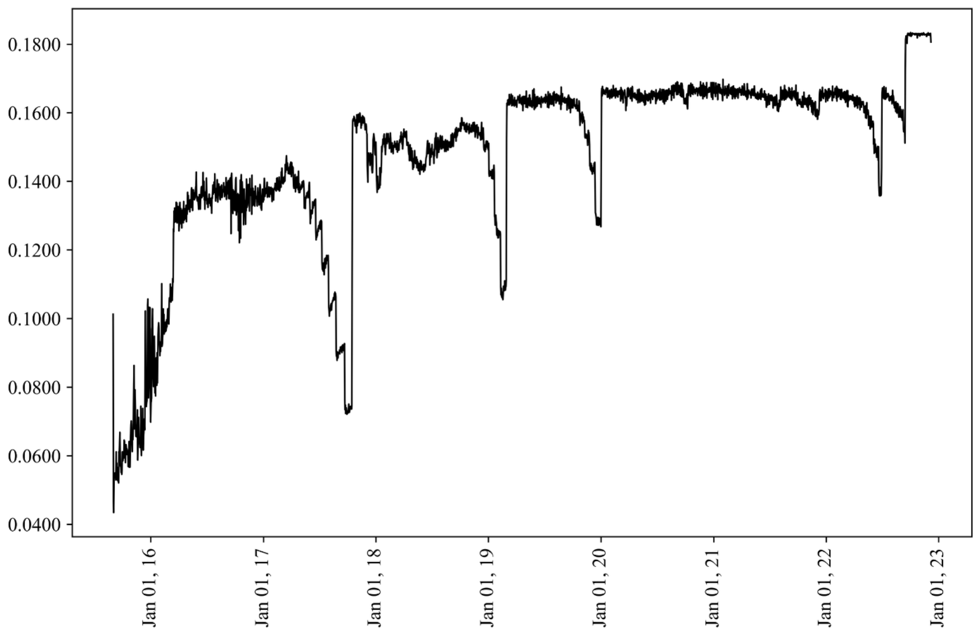

Fig. 6. TPPF over time.

Fig. 6 offers a visual analysis of the utility over time, as interpreted through the application of the arctan utility function. This analysis reveals progressive improvement in the network's TPPF over time, highlighting the efficacy of employing the arctan utility function as a measure of evaluation.

5.4. Comparisons of the Extensions and Baseline Model

As displayed in Table 4, the evaluation of adding extensions to the baseline model, which employs both nonlinear and linear regression, incorporates the value drivers of quality and utility. This integration not only captures the nuanced quality of the network but also reinforces the model's enhanced accuracy and reliability. Notably, quality (TCPU) demonstrates superior predictive power, followed by utility (TPPF), as evidenced by a higher R-squared coefficient and a lower RMSD. Furthermore, incorporating the extensions exhibits superior predictive capabilities when compared to the R-squared and RMSD values of the previous baseline model, which primarily considered only network size. This underscores the model's enhanced precision within the regression analysis framework. Therefore, a more accurate pricing model appears to be the baseline model, which accounts for the size of the network and is enhanced by extensions that improve our valuation of the Ethereum Network's price, defined by quality and utility improvements within the network.

Table 4. Regression model outputs for the extensions.

Regression	**Extensions**	**size (baseline model)**	**quality (TCPU)**	**utility (TPPF)**	**RMSD**	**R^2**
Linear regression	a) size	18.7990			0.6274	0.9127
	b) size & quality	11.0599	0.3467		0.4761	0.9497
	c) size & utility	18.7636		0.1885	0.6274	0.9128
	e) size, quality & utility	10.4095	0.3516	2.8762	0.4734	0.9503
Non-linear regression	a) size				0.2763	0.9801
	b) size & quality				0.1852	0.9915
	c) size & utility				0.1892	0.9911
	e) size, quality & utility				0.1654	0.9932

5.5. Training and Testing Extensions

To evaluate the extended regression model's predictive accuracy on novel data and mitigate the risk of overfitting, the dataset was systematically divided into training and testing subsets. The observed congruence between the R-squared values from both the training and testing phases suggests a negligible likelihood of overfitting. Furthermore, the closeness of the RMSD values for both training and testing phases reinforces this inference. The similarity of these metrics, taken together, signifies that the model with the extensions demonstrates robust generalizability to unseen data. As a result, the baselined model appears to have passed the quality test in a decentralized network, Ethereum. In light of these findings, the extensions clearly enhance the baseline model for evaluating the Ethereum Network, indicating that the price of the Ethereum network is influenced not only by its size but also by the quality and utility of the network. Consequently, we propose a model for valuing the Ethereum Network that utilizes non-linear regression, employing artificial neural network models that account for size of the Network as the baseline model and incorporate two independent variables representing quality and utility improvements of the network (TCPU and TPPF). The architecture of the ANN model is encapsulated in Eq. (17).

Future research could be motivated by the analysis of applying the proposed Ethereum network model to other Layer 1 blockchain networks, such as Bitcoin, Avalanche and Near. Comparing these outcomes with the predictive capabilities of previously published valuation methods, particularly those aligned with Metcalfe's Law, could assist in developing a comprehensive formula based on network effects theory that fully captures the value within the Layer 1 blockchain space. This avenue appears to merit further investigation for a more nuanced understanding of Layer 1 blockchain network valuation.

6. Conclusion

The proposes model extends Ref. [1] demand-based model by incorporating adjustments for the circulation supply of the Ethereum Network and incorporating non-linear regression functions to capture complex non-linear patterns of demand and supply ratio at smaller time scales while following the overall trend. In this regression framework, two independent variables have been formulated and integrated to capture the network's quality and utility (TCPU and TPPF), variables that exceed rudimentary network size considerations. As a result, the model appears to effectively captures the past and future upgrades of the Ethereum Network.

The proposed model demonstrates a strong fit to the data, significantly increasing its explanatory power. This extended model represents a notably important contribution to the standard model proposed by Ref. [1] for valuing the Ethereum Network, based on network effects value (size), quality & utility (inherently tied to quality), and the fundamental principle of the law of supply and demand. Furthermore, this model enables new opportunities for application to other Layer 1 blockchain networks.

Appendix

A Non-linear regression

In this section, we provide a detailed explanation of the non-linear regression introduced in Section 3.3.

A.1. Model architecture of the learning algorithm

The architecture of the Hybrid Recurrent-Convolutional Neural Network with an Attention Mechanism is designed as follows:

Algorithm 1: Model Architecture

Input: Training samples X_t, target samples Y_{t+a}, window length α

Output: Price prediction $\hat{Y}_{t+a}$

1 Initialize GRU layer to establish preliminary memory;
2 Apply MLP with a single hidden layer for attention mechanism;
3 Use Dense layer as a bridge between MLP and CNN;
4 CNN layer extracts local and global features;
5 Final Dense layer outputs the predicted value;
6 Optimize model parameters using Adam optimizer;

The learning algorithm initially defines a function to create sequences, thereby transforming the time series data into a format suitable for training a Recurrent Neural Network (RNN). This function generates sequences of a specified length, known as a window, from the input data. Each sequence is then used to predict the subsequent value in the time series. The normalized data is subsequently transformed into sequences of a specific length to create the final input data, denoted as X_{seq}, and the target data, denoted as $Y_t + a$. Activation functions, including Rectified Linear Unit ($ReLU$), sigmoid (σ), and hyperbolic tangent function (tanh), have been deployed to enable non-linear transformations.

A.2. Layers of the learning algorithm

A.2.1. GRU layer

The Gated Recurrent Unit (GRU) layer is computed as depicted in (20), effectively captures both short term and long-term dependencies within the data. The GRU layer processes the input data, X_{seq}, and applies the transformations. The terms R_t, Z_t, $\tilde{h}_t$ and h_t are computed using the σ and tanh activation functions, respectively. The weights and biases are represented by W_{rx}, W_{rs}, W_{zx}, W_{zs}, W_{hx}, W_{hs}, b_r, b_z, and b_h. Additionally, the symbols $h_{t-1}$and $\tilde{h}_t$ signify the previous hidden state and current hidden state, respectively. The element-wise multiplication operation is represented by Eq. (20).

$$
\begin{aligned}
R_t &= \sigma\left(W_{rx} \cdot X_{seq} + W_{rs} \cdot h_{t-1} + b_r\right) \\
Z_t &= \sigma\left(W_{zx} \cdot X_{seq} + W_{zs} \cdot h_{t-1} + b_z\right) \\
\tilde{h}_t &= \tanh\left(W_{hx} \cdot X_{seq} + W_{hs} \cdot (R_t \odot h_{t-1}) + b_h\right) \\
h_t &= (1 - Z_t) \odot h_{t-1} + Z_t \odot \tilde{h}_t
\end{aligned} \tag{20}
$$

A.2.2. MLP layer

The attention mechanism is implemented as a Multilayer Perceptron (MLP) layer as described in Ref. (21) and is deployed to weigh and discern the significance of different time steps in the input sequence. The attention mechanism is applied to the output of the GRU layer, represented by h_t. The tanh function is applied to the weighted sum of the hidden states using the weights W_a and bias b_a. The softmax function is then applied to obtain the attention weights, denoted as A_t, which represent the importance of each time step in the sequence.

$$
\begin{aligned}
a_t &= \tanh(W_a h_t + b_a) \\
A_t &= \mathrm{softmax}(a_t)
\end{aligned} \tag{21}
$$

A.2.3. Dense layer

The Dense layer, as expressed in Eq. (22), serves as a bridge between the attention mechanism MLP and the Convolutional Neural Network (CNN) layer. It transforms the output of the attention mechanism into a format suitable for the CNN layer. In this transformation, the Dense layer applies a mathematical function known as the tanh activation function to the output of the attention mechanism, denoted as A_t. W_d and b_d represent the weights and bias of the Dense layer respectively, and d_t is the output of the Dense layer.

$$
d_t = \tanh(W_d A_t + b_d) \tag{22}
$$

A.2.4. CNN layer

The CNN layer, described in Eq. (23), is employed to extract local and global features from the input sequence. Specifically, the Conv1D layer is applied to the output of the Dense layer, represented as d_t. The $*$ operation represents the convolution operation. The ReLU function is applied to the weighted sum of the inputs using the weights W_c and bias b_c. The resulting output, c_t, is obtained. The ReLU function introduces non-linearity into the CNN layer, enabling it to capture complex patterns and relationships in the data. This approach potentially offers the best of both worlds, allowing the modeling of the non-linear relationship between the feature and target variable with the CNN layer while considering the temporal nature of the data with the GRU component. This approach outperforms other methods based on the results of the study by Ref. [9].

$$
c_t = \mathrm{ReLU}(W_c \times d_t + b_c) \tag{23}
$$

A.2.5. Dense layer

The final Dense layer, as presented in Eq. (24), generates the model's prediction. The output layer is applied to the output of the Conv1D layer, denoted as W_y . c_t and b_y represent the weights and bias of the output layer, respectively, and y_t is the final output of the model.

$$y_t = W_y c_t + b_y \tag{24}$$

A.3 Parameter Settings of the Learning Algorithm

Table 5. Parameter configurations for each component of the learning algorithm, including GRU, CNN, and attention-enhanced MLP layers used in the nonlinear model architecture

Layer	Parameter settings	Value
GRU	Neurons	32
	Layers	1
	Activation function	tanh
MLP: attention mechanism	MLP neurons	1
	MLP layers	1
	MLP activation function	tanh
	Attention mechanism activation function	softmax
Dense layer	Neurons	1
	Layers	1
	Activation function	tanh
CNN layer	Filters	32
	Kernel Size	3
	Activation function	ReLU
	Dimensional convolutional layers	1
	Regularization: l1 & l2	0.01
Final Dense layer	Neurons	1
	Layers	1

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Ken Alabi provided methodological guidance, supervised the research direction, analyzed the data, and validated the findings. Joshua Eick conducted the research, carried out the methodology and investigation, contributed to the conceptualization and implementation, and wrote the original draft of the manuscript. All authors have reviewed and approved the final version of the manuscript.

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